What is a Meta-Analysis?

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Outline

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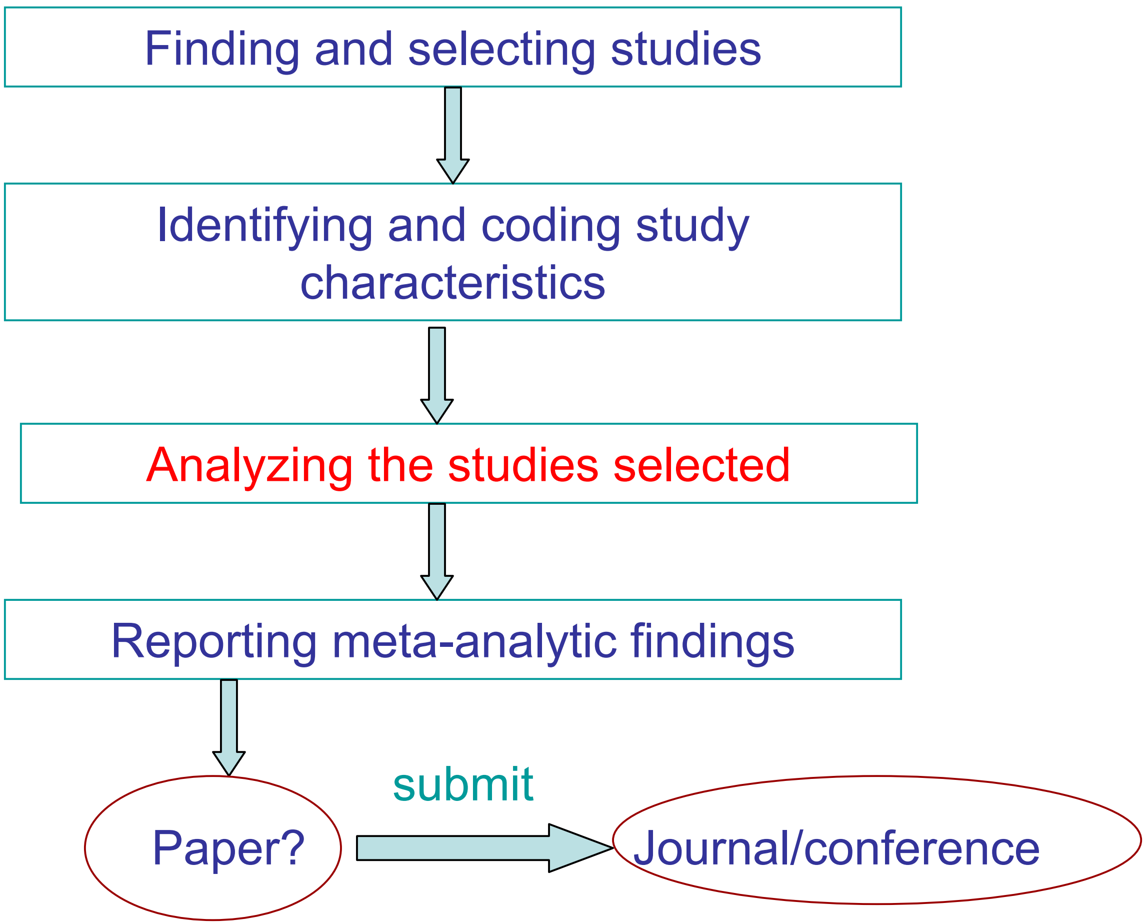
What is a meta-analysis

- In 1976, Glass coined the term meta-analysis
- *Meta-analysis refers to the analysis of analyses… the **statistical analysis** of a large collection of **analysis results** from individual studies for the purpose of integrating the findings.* (Glass, 1976, p3)
- Meta-analysis techniques are needed because only summary statistics are typically available in the literature.
- Often used in medical and psychological studies.

Literature reviews vs Meta-analysis

Aims	To examine the relevant literature for general trends and patterns	
	Literature review	**Meta-analysis**
Methods (combining the results from independent studies)	qualitatively with little, if any, quantitative manipulation of the published information	statistical analysis
Results (quality)	Subjective, scientifically unsound	more accurate

Guidelines for meta-analysis



meta-analytical statistical methods

- Combining significance tests (p value).
- Concerning differences between means, i.e. those dealing with effect sizes.
- Dealing with relationships between variables, i.e. those concerning correlations.

Combine significance tests

- Stouffer-method (Rosental, 1984)
- Based on adding the standard normal deviation Z.
- Procedure: (Pi is available)
  - Pi->Zi
  - $$Z(overall) = \frac{\sum Zi}{\sqrt{K}}$$
  - Z (overall)-→ P(overall)
- This P (overall) yields an overall level of significance.

Combined significance tests -cont.

- **Advantage:** lies in the increased power of the overall comparison.
  - If several tests consistently favor the research question but fail to reach the level of significance, due to small sample sizes, the overall test would more easily become significant because the pooled sample size is much larger.
- **Procedure that combine significance tests have been criticized for:**
  - The focus is on an overall probability instead of on distributions.
  - A *P* value just indicates the probability of an error in rejecting the null hypothesis if it were true.
  - It does not provide, however, an estimate of the magnitude of effects.

Combination of effect sizes

- **Effect size** (d) :
  - a standardized difference between a pair of means of experimental and a control groups.
  - the strength of a relationship between the treatment and the outcome variable (the magnitude of treatment effects).
  - Categories: 0.2->small, 0.5->medium, 0.8->Large (Cohen, 1977).
- **Procedure:**
  - **Calculate** effect sizes: two means (M), SD, N
  - **Combine** effect sizes across studies

Calculation of the effect size

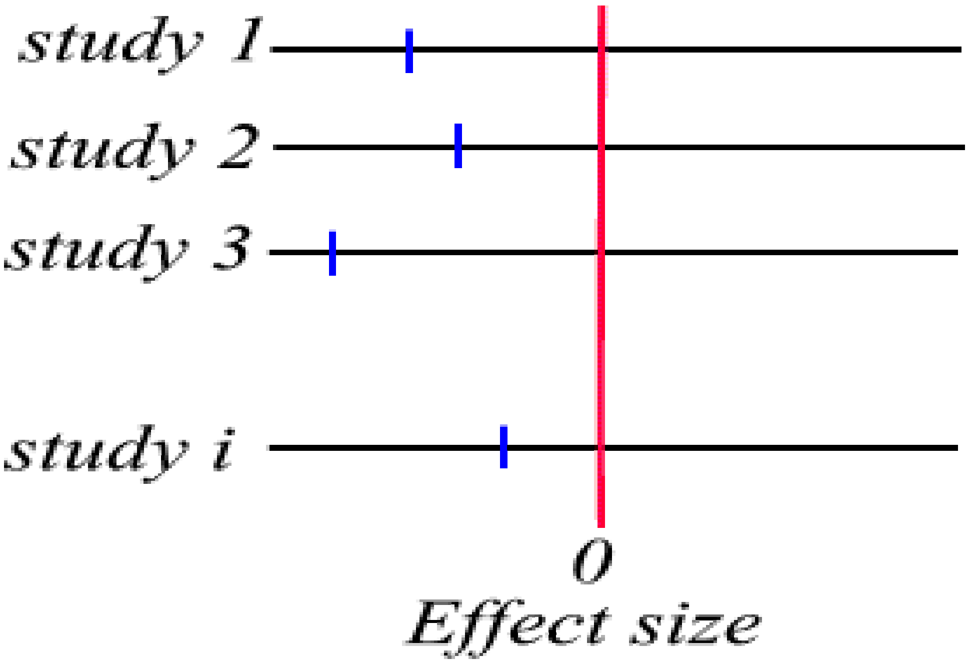
$$d_i = \frac{\overline{x}_i^E - \overline{x}_i^C}{s_i} J$$

$$s_i = \sqrt{\frac{(N_i^E - 1)(s_i^E)^2 + (N_i^C - 1)(s_i^C)^2}{N_i^E + N_i^C - 2}}$$

$$J = 1 - \frac{3}{4(N_i^E + N_i^C - 2) - 1}$$

- Where X-barE and X-bar C are the experimental and control group means
- Si –the pooled standard deviation
- J is the correction for small and unequal sample sizes.

Graphical presentation



Combining effect sizes across studies

- Decide to if a *fixed, random or mixed model* is appropriate for the selected studies.
  - Fixed: the true magnitude of the effect (delta) is assumed to be a constant, whose value is unknown but is estimated by the *di* values from the included studies. (early)
  - Random: delta can vary between studies
    - the variance of a sample *di* is conditional on the value of *deltai.*
    - Using random effect model is relatively rare
  - Mixed: there is random variation between studies in a class but that they are share a common mean effect.
    - Most useful.

Fixed effect analysis

$$\overline{d} = \frac{\sum w_i d_i}{\sum w_i}$$

$$s_d^2 = \frac{1}{\sum w_i}$$

$$w_i = 1 / v_i$$

$$v_i = \frac{N_i^E + N_i^C}{N_i^E N_i^C} + \frac{d_i^2}{2(N_i^E + N_i^C)}$$

- Calculate confidence limits (95%)
  - Upper limit = mean + (z(0.05,2) . sd)
  - Lower limit = mean - (z(0.05,2) . sd)
  - z(0.05,2) is a 2-tailed value of z (1.96).
- d-bar: overall mean effect size; wi: weights; vi: variance ; s2: variance for the accumulated effect sizes.

Gurevitch and Hedges (1993) report the results of a met-analysis of competition in three groups of primary producers.

Habitat	number of studies	mean effect size	s_d	95% CL
Terrestrial	19	1.15	0.118	0.919 - 1.375
Lentic	2	4.11	0.885	2.371 - 5.843
Marine	21	0.80	0.122	0.557 - 1.040
Overall	42	1.01*	0.084	0.836 - 1.174

- In all three cases, the mean effect size is > 0 . so there is evidence that competition affects the response variable (either no. of individual or size).
- There is also evidence that the size of the effect differs between habitats, being much larger for lentic plants.
- Overall there is evidence for an effect of competition on plants in the three habitats

Combining correlation coefficients (Hunter, 1982)

- Correlation coefficient (*r*)
  - A single number that describes both the strength and the direction of the relationship.
  - -1~+1(r=0, no relationship; r=1.0, perfect positive correlation; r=-0.6, weak negative correlation)
- Eliminate the bias caused by:
  - Sampling error: the random variation in study results due to sample size (major source of variation:10-100%);
  - Measurement error
  - Effects of moderator variables

Sampling Error

- If it is reasonable to assume that the studies are estimating a common population correlation coefficient (r), a combined estimate can be made.
- However, if the sample sizes differ the estimate should not be a simple mean.
- As with all statistics, sample correlations based on large sample sizes are more reliable.
- This must be taken into account by weighting the average with respect to sample sizes.

$$\overline{r} = \frac{\sum [N_i r_i]}{\sum N_i}$$

$$S_r^2 = \frac{\sum [N_i (r_i - \overline{r})^2]}{\sum N_i}$$

A simple example

r	*n*
0.75	20
0.32	120
0.62	60

- Assume that we have 3 sample correlation coefficients. What is their mean?
- The simple average correlation coefficient is 0.563
- weighted mean r

  =[(20 * 0.75) + (120 * 0.32) + (60 * 0.62)] / (20 + 120 +60)

  = 0.453
- because of the low correlation coefficient in the largest sample.
- variance =[20(0.75 - 0.453)2 + 120(0.32 - 0.453)2 + 60(0.62 - 0.453)2] / 200 =0.0395

Sampling error –cont.

- Hunter (1982) demonstrated that the variance across studies is the sum of two components:
  - the variance due to sampling error =3 [(1 - 0.4532)2] / 200 =0.00316
  - the variance in the population correlations (range variation): 0.0395 - 0.00316 = 0.0363 !=0
  - The population correlation coefficients differ across studies.
  - A significance test:
    - ! it is very powerful and may reject H0 even a trivial variation

$$s_e^2 = \frac{k(1-\bar{r}^2)^2}{\sum N_i}$$

$$s_p^2 = s_r^2 - s_e^2$$

$$x_{k-1}^2 = \frac{\sum N_i}{(1-\bar{r}^2)^2} s_r^2$$

$$\frac{s_e^2}{s_r^2} < 75\%$$

Moderator variable may be present

Dealing with moderator variables

Group	n	r
1	100	0.34
1	100	0.16
1	50	0.12
1	50	0.38
2	100	0.19
2	100	0.01
2	50	-0.03
2	50	0.23

r-bar (mean r)	0.175
s^2_r	0.0167
s^2_e	0.0125
s^2_p	0.0042 (0.0167 - 0.0125)
s_p	0.065

P=0.058

r-bar (mean r)	0.25	0.10
s^2_r	0.011	0.011
s^2_e	0.0117	0.0117
s^2_p	-0.0007	-0.0007
s_p	0	0

Analyze the sub-groups independently

Ref: Hunter(1982, p49)

Measurement errors

- Hunter describes a technique (not described here) for correcting the bias in sample correlation coefficients caused by measurement error.
- Unfortunately the method makes use of measurement reliability indices that will usually be unavailable.

reference

- Glass, G. V. 1976. Primary, secondary and meta-analysis of research. *Educational Researcher*, 5: 351-379.
- Gurevitch, J. and Hedges, L. V. 1993. Meta-analysis: combining the results of independent experiments. Pp 378-425 *in*: S. M. Scheiner and J. Gurevitch (eds) *Design and analysis of ecological experiments*. Chapman and Hall, London.
- Hunter, J. E. 1982. *Meta Analysis*. Sage Publications, Beverly Hills.
- http://149.170.199.144/resdesgn/meta.htm

Meta-Analysis Links

- Cooper and Hedges, 1994 (a comprehensive multi-author text)
- Bailar, 1995 (a 'demolition-job' on 5 meta-analyses)
- Gurevitch and Hodges, 1993 (ecological met-analysis); , 1993 (general);
- Hedges and Olkin, 1985 (general text);
- Jones, 1995 (general review);
- Rosenthal, 1995 (general guidelines).
- Handbook for Research Synthesis Cooper & Hedges

Links to Meta-Analysis Software

- **Commercial**
  - Comprehensive Meta-Analysis: www.meta-analysis.com
  - MetaWin: www.metawinsoft.com
  - WEasyMa: www.weasyma.com
- **Free-ware**
  - RevMan (Review Manager) developed by the Cochrane Collaboration: www.cochrane.org
  - Meta-Analysis Version 5.3: www.statistics.com/content/freesoft/mno/meta-ana53.html
- **Statistics software with meta-analytic tools or macros**
  - SAS: www.sas.com
  - STATA: www.stata.com
  - WinBUGS: www.mrc-bsu.cam.ac.uk/bugs
    - WinBUGS is a package of programs for Bayesian analysis and can also be used for Bayesian meta-analysis
- **Links to other meta-analysis software**
  - www.prw.le.ac.uk/epidemio/personal/ajs22/meta
  - http://epiweb.massey.ac.nz/meta_analysis_software.htm