Non-linear Bone Adaptation Simulation
Toward
Uniform Strain Energy Density

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Bone Adaptation

● Biological tissues have the unique property that cells can alter tissue matrix in response to external environmental stimuli.

● It is believed that cells within the tissues can sense some aspect of the mechanical stimulus, judge whether this stimulus is appropriate, and then alter the tissue matrix in an appropriate manner to respond to the mechanical stimulus (Cowin, 1990).
Bone Adaptation

Clinical and health implications include:

- If the adaptation capability becomes unbalanced…
  diseases like osteoporosis appear

- Stress shielding around artificial joints:-

Since the artificial joint metal is an order of magnitude stiffer than bone, the strain in the surrounding bone is significantly reduced below normal. This leads to resorption of bone tissue around the artificial joint.

Can this adaptation capability be predicted?
Wolff’s Law (1870-1894)

"Every change in the form and the function of a bone or of their function alone is followed by certain definite changes in their internal architecture, and equally definite secondary alterations in their external confirmation, in accordance with mathematical laws"

• Wolff was a German Anatomist who is credited with general theory of bone adaptation known as Wolff's Law: 1870-1894
• Suggested that bone obtained maximum mechanical efficiency with minimum mass -> optimal
• Bone structure could adapt in response to changing Mechanical Environment
Modern Numerical Computational Model

1. Start with initial bone structure
2. Update bone structure
3. Update effective bone stiffness based on updated bone structure
4. Solve equilibrium equation for new bone stiffness to determine new SED* field
5. If not converge yet, return to step 2; otherwise, go to step 6
6. Stop with convergence

*SED: Strain Energy Density

Fig. 2: Flow chart of numerical model
Bone Structure Updating

The non-linear bone adaptation model was originally proposed in 1992 as

\[ \frac{d\rho(x,t)}{dt} = B(t)\left(\sum_{i=1}^{N} f_i(x)(\beta_i^\alpha - 1)\right), 0 < \rho \leq \rho_{cb} \]

\[ \beta_i = \frac{U_a(i)}{\rho_i k} \]  -- Comparative coefficient

\[ f_i(x) \]  -- Spatial influencing function,

\[ B(t) \]  -- Time-dependant remodeling coefficient

\[ \rho \]  -- Apparent bone density

\[ \alpha \]  -- Order of non-linear remodeling equation
The Numerical Computational Model @ BII

Features:
• Non-linear bone structure updating
• Nodal Formulation
• Generalized Conjugate Residue (GCR) solver
• Computationally less expensive

Fig. 3: Flow chart of numerical model (on the right)
Generalized Conjugate Residue Solver

Why GCR Solver? Compare:

<table>
<thead>
<tr>
<th>Solver</th>
<th>LU</th>
<th>QR</th>
<th>GCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complexity</td>
<td>O(N³)</td>
<td>O(N³)</td>
<td>O(N²) Due to properties of G</td>
</tr>
</tbody>
</table>

Using Krylov subspace, GCR solver can solve the system with $O(N^2)$ operations due to advantageous properties of matrix $G$. 
Comparison

- CT from Visible Human Project
- 1867: Von Meyer a German anatomist
- Simulation Result
- Simulation Result

Fig. 6: Comparison of Results
Factors Affecting the Result

• Model Parameters:-
  \( B \): Remodeling coefficient
  \( \alpha \): Non-linearity
  \( K \): normalizing constant

• Loading data:-
  not easy to measure accurately

• Mesh density:-
  depends on computing power

• Others:-
  biological understanding of bone adaptation mechanism…
**Future Work**

Future work may include:

- Develop parallel version of bone adaptation simulation program for large scale computing
- Fine-tune the non-linear bone remodeling model (model parameters, etc)
- Get more proper data of femur loading during daily activities (gait analysis)
- Compare Nodal Formulation vs FEM (computing performance, cost, accuracy, etc)
- 3D large scale bone adaptation simulation
- Integrate the computing results with clinical data for a better understanding of bone remodeling, mechanisms of osteoporosis
References

• Jacob White, SMA5211 lecture notes, SMA-HPCES, Singapore-MIT Alliance 2001;


• Computational Modeling of Biological Tissues, U of Michigan at http://www.engin.umich.edu/class/bme506/bme5062000/bme506formlec/compadapt/boneadapt.htm